



# Mathematics HL subject outline

First examinations 2014



This document explains the major features of the course, and outlines the syllabus and assessment requirements.

More detailed information about the course can be obtained by referring to the guide for this subject, which is available on the subject page of the IB online curriculum centre (OCC) website (<http://occ.ibo.org>) and can also be purchased from the IB store (<http://store.ibo.org>).

# Nature of the subject

## Introduction

The nature of mathematics can be summarized in a number of ways: for example, it can be seen as a well-defined body of knowledge, as an abstract system of ideas, or as a useful tool. For many people it is probably a combination of these, but there is no doubt that mathematical knowledge provides an important key to understanding the world in which we live. Mathematics can enter our lives in a number of ways: we buy produce in the market, consult a timetable, read a newspaper, time a process or estimate a length. Mathematics, for most of us, also extends into our chosen profession: visual artists need to learn about perspective; musicians need to appreciate the mathematical relationships within and between different rhythms; economists need to recognize trends in financial dealings; and engineers need to take account of stress patterns in physical materials. Scientists view mathematics as a language that is central to our understanding of events that occur in the natural world. Some people enjoy the challenges offered by the logical methods of mathematics and the adventure in reason that mathematical proof has to offer. Others appreciate mathematics as an aesthetic experience or even as a cornerstone of philosophy. This prevalence of mathematics in our lives, with all its interdisciplinary connections, provides a clear and sufficient rationale for making the study of this subject compulsory for students studying the full diploma.

## Summary of courses available

Because individual students have different needs, interests and abilities, there are four different courses in mathematics. These courses are designed for different types of students: those who wish to study mathematics in depth, either as a subject in its own right or to pursue their interests in areas related to mathematics; those who wish to gain a degree of understanding and competence to understand better their approach to other subjects; and those who may not as yet be aware how mathematics may be relevant to their studies and in their daily lives. Each course is designed to meet the needs of a particular group of students. Therefore, great care should be taken to select the course that is most appropriate for an individual student.

In making this selection, individual students should be advised to take account of the following factors:

- their own abilities in mathematics and the type of mathematics in which they can be successful
- their own interest in mathematics and those particular areas of the subject that may hold the most interest for them
- their other choices of subjects within the framework of the Diploma Programme
- their academic plans, in particular the subjects they wish to study in future
- their choice of career.

Teachers are expected to assist with the selection process and to offer advice to students.

## Mathematical studies SL

This course is available only at standard level, and is equivalent in status to mathematics SL, but addresses different needs. It has an emphasis on applications of mathematics, and the largest section is on statistical techniques. It is designed for students with varied mathematical backgrounds and abilities. It offers students opportunities to learn important concepts and techniques and to gain an understanding of a wide variety of mathematical topics. It prepares students to be able to solve problems in a variety of settings, to develop more sophisticated mathematical reasoning and to enhance their critical thinking. The individual project is an extended piece of work based on personal research involving the collection, analysis and evaluation of data. Students taking this course are well prepared for a career in social sciences, humanities, languages or arts. These students may need to utilize the statistics and logical reasoning that they have learned as part of the mathematical studies SL course in their future studies.

## Mathematics SL

This course caters for students who already possess knowledge of basic mathematical concepts, and who are equipped with the skills needed to apply simple mathematical techniques correctly. The majority of these students will expect to need a sound mathematical background as they prepare for future studies in subjects such as chemistry, economics, psychology and business administration.

## Mathematics HL

This course caters for students with a good background in mathematics who are competent in a range of analytical and technical skills. The majority of these students will be expecting to include mathematics as a major component of their university studies, either as a subject in its own right or within courses such as physics, engineering and technology. Others may take this subject because they have a strong interest in mathematics and enjoy meeting its challenges and engaging with its problems.

## Further mathematics HL

This course is available only at higher level. It caters for students with a very strong background in mathematics who have attained a high degree of competence in a range of analytical and technical skills, and who display considerable interest in the subject. Most of these students will expect to study mathematics at university, either as a subject in its own right or as a major component of a related subject. The course is designed specifically to allow students to learn about a variety of branches of mathematics in depth and also to appreciate practical applications. It is expected that students taking this course will also be taking mathematics HL.

**Note:** Mathematics HL is an ideal course for students expecting to include mathematics as a major component of their university studies, either as a subject in its own right or within courses such as physics, engineering or technology. It should not be regarded as necessary for such students to study further mathematics HL. Rather, further mathematics HL is an optional course for students with a particular aptitude and interest in mathematics, enabling them to study some wider and deeper aspects of mathematics, but is by no means a necessary qualification to study for a degree in mathematics.

## Mathematics HL—course details

The course focuses on developing important mathematical concepts in a comprehensible, coherent and rigorous way. This is achieved by means of a carefully balanced approach. Students are encouraged to apply their mathematical knowledge to solve problems set in a variety of meaningful contexts. Development of each topic should feature justification and proof of results. Students embarking on this course should expect to develop insight into mathematical form and structure, and should be intellectually equipped to appreciate the links between concepts in different topic areas. They should also be encouraged to develop the skills needed to continue their mathematical growth in other learning environments.

The internally assessed component, the exploration, offers students the opportunity for developing independence in their mathematical learning. Students are encouraged to take a considered approach to various mathematical activities and to explore different mathematical ideas. The exploration also allows students to work without the time constraints of a written examination and to develop the skills they need for communicating mathematical ideas.

This course is a demanding one, requiring students to study a broad range of mathematical topics through a number of different approaches and to varying degrees of depth. Students wishing to study mathematics in a less rigorous environment should therefore opt for one of the standard level courses, mathematics SL or mathematical studies SL. Students who wish to study an even more rigorous and demanding course should consider taking further mathematics HL in addition to mathematics HL.

## Prior learning

Mathematics is a linear subject, and it is expected that most students embarking on a Diploma Programme (DP) mathematics course will have studied mathematics for at least 10 years. There will be a great variety of topics studied, and differing approaches to teaching and learning. Thus students will have a wide variety of skills and knowledge when they start the mathematics HL course. Most will have some background in arithmetic, algebra, geometry, trigonometry, probability and statistics. Some will be familiar with an inquiry approach, and may have had an opportunity to complete an extended piece of work in mathematics.

At the beginning of the syllabus section there is a list of topics that are considered to be prior learning for the mathematics HL course. It is recognized that this may contain topics that are unfamiliar to some students, but it is anticipated that there may be other topics in the syllabus itself that these students have already encountered. Teachers should plan their teaching to incorporate topics mentioned that are unfamiliar to their students.

## Links to the Middle Years Programme

The prior learning topics for the DP courses have been written in conjunction with the Middle Years Programme (MYP) mathematics guide. The approaches to teaching and learning for DP mathematics build on the approaches used in the MYP. These include investigations, exploration and a variety of different assessment tools.

A continuum document called *Mathematics: The MYP–DP continuum* (November 2010) is available on the DP mathematics home pages of the OCC. This extensive publication focuses on the alignment of mathematics across the MYP and the DP. It was developed in response to feedback provided by IB World Schools, which expressed the need to articulate the transition of mathematics from the MYP to the DP. The publication also highlights the similarities and differences between MYP and DP mathematics, and is a valuable resource for teachers.

# Aims

## Group 5 aims

The aims of all mathematics courses in group 5 are to enable students to:

1. enjoy mathematics, and develop an appreciation of the elegance and power of mathematics
2. develop an understanding of the principles and nature of mathematics
3. communicate clearly and confidently in a variety of contexts
4. develop logical, critical and creative thinking, and patience and persistence in problem-solving
5. employ and refine their powers of abstraction and generalization
6. apply and transfer skills to alternative situations, to other areas of knowledge and to future developments
7. appreciate how developments in technology and mathematics have influenced each other
8. appreciate the moral, social and ethical implications arising from the work of mathematicians and the applications of mathematics
9. appreciate the international dimension in mathematics through an awareness of the universality of mathematics and its multicultural and historical perspectives
10. appreciate the contribution of mathematics to other disciplines, and as a particular “area of knowledge” in the TOK course.

# Assessment objectives

Problem-solving is central to learning mathematics and involves the acquisition of mathematical skills and concepts in a wide range of situations, including non-routine, open-ended and real-world problems. Having followed a DP mathematics HL course, students will be expected to demonstrate the following.

1. **Knowledge and understanding:** recall, select and use their knowledge of mathematical facts, concepts and techniques in a variety of familiar and unfamiliar contexts.
2. **Problem-solving:** recall, select and use their knowledge of mathematical skills, results and models in both real and abstract contexts to solve problems.
3. **Communication and interpretation:** transform common realistic contexts into mathematics; comment on the context; sketch or draw mathematical diagrams, graphs or constructions both on paper and using technology; record methods, solutions and conclusions using standardized notation.
4. **Technology:** use technology, accurately, appropriately and efficiently both to explore new ideas and to solve problems.
5. **Reasoning:** construct mathematical arguments through use of precise statements, logical deduction and inference, and by the manipulation of mathematical expressions.
6. **Inquiry approaches:** investigate unfamiliar situations, both abstract and real-world, involving organizing and analysing information, making conjectures, drawing conclusions and testing their validity.

# Syllabus outline

Syllabus component	Teaching hours
	HL
All topics are compulsory. Students must study all the sub-topics in each of the topics in the syllabus as listed in this guide. Students are also required to be familiar with the topics listed as prior learning.	
<b>Topic 1</b> Algebra	30
<b>Topic 2</b> Functions and equations	22
<b>Topic 3</b> Circular functions and trigonometry	22
<b>Topic 4</b> Vectors	24
<b>Topic 5</b> Statistics and probability	36
<b>Topic 6</b> Calculus	48
<b>Option syllabus content</b> Students must study all the sub-topics in one of the following options as listed in the syllabus details. <b>Topic 7</b> Statistics and probability <b>Topic 8</b> Sets, relations and groups <b>Topic 9</b> Calculus <b>Topic 10</b> Discrete mathematics	48
<b>Mathematical exploration</b> Internal assessment in mathematics HL is an individual exploration. This is a piece of written work that involves investigating an area of mathematics.	10
<b>Total teaching hours</b>	<b>240</b>

# Assessment outline

## First examinations 2014

Assessment component	Weighting
<b>External assessment (5 hours)</b> <b>Paper 1 (2 hours)</b> No calculator allowed. (120 marks) <i>Section A</i> Compulsory short-response questions based on the core syllabus. <i>Section B</i> Compulsory extended-response questions based on the core syllabus.	<b>80%</b> <b>30%</b>
<b>Paper 2 (2 hours)</b> Graphic display calculator required. (120 marks) <i>Section A</i> Compulsory short-response questions based on the core syllabus. <i>Section B</i> Compulsory extended-response questions based on the core syllabus.	<b>30%</b>
<b>Paper 3 (1 hour)</b> Graphic display calculator required. (60 marks) Compulsory extended-response questions based mainly on the syllabus options.	<b>20%</b>
<b>Internal assessment</b> This component is internally assessed by the teacher and externally moderated by the IB at the end of the course. <b>Mathematical exploration</b> Internal assessment in mathematics HL is an individual exploration. This is a piece of written work that involves investigating an area of mathematics. (20 marks)	<b>20%</b>