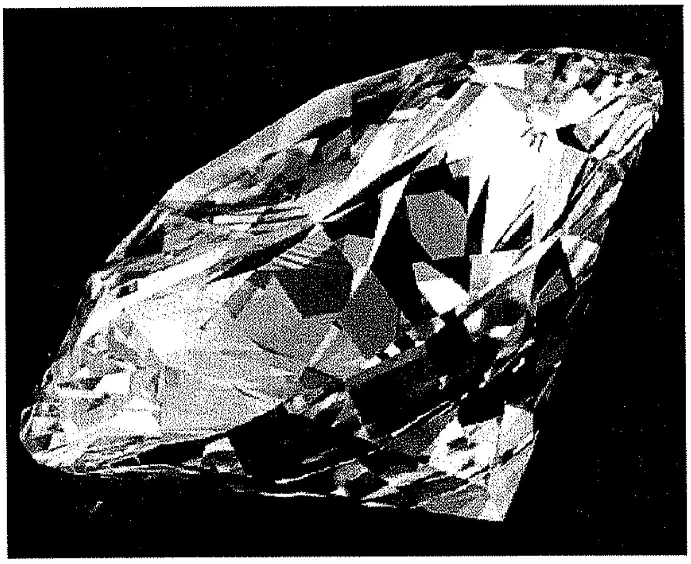
Math Project

The diamond ideal cut



Subject: Mathematics

Examination year: May 2009

Outline

A. **Introduction**

For my math project I have chosen a topic that I am familiar with and that I am very fond off: diamonds. Thanks to my father who is a diamond dealer and expert, I have been introduced to diamonds since a young age. In this paper, I have focused on two problems.

Various sources that I have researched presented a table in which a mass of a diamond is related to its diameter. My first question is if the table can be justified by mathematical methods. The table I took under consideration is (Table 1)[1] referred to a 2D version of the diamond 'ideal cut' a cut developed by the famous polisher Martin Tolkowsky. As it will be shown the shape of a polished diamond can be divided into simple geometric solids. They give a good approximation of the volume of a diamond. The volume multiplied by the density of a diamond should give the result close to those in the table. I will calculate the correlation between my results and those in the table to evaluate how close they are.

The second problem goes over the lost parts of a diamond. For the idea of finding the lost parts of a rough diamond came to me as I was going to the polishing office with my father and I realized how much diamond powder was on the floor, I felt that it was wasted. While polishing a part of the diamond a lot of the rough's weight mass is wasted. To illustrate how big that amount is I investigated one of the largest diamond ever found, named 'the Cullinan'. Besides that I will try to find a relationship between the initial (rough) weight and the final (polished) one. For that purpose I have collected data of weights of diamonds before and after polishing. This will allow me to evaluate what is the mass of a polished diamond when the mass of a new one is given. I have attempted to find a relationship between the amount lost and the initial weight by using regression lines.

A2

Table 1 A set that shows the relationship between diamond weight and diameter

Weight: ct	Size mm: (diameter)
0.05	1.00
0.10	3.00
0.20	3.85
0.25	4.10
0.33	4.55
0.40	4.80
0.50	5.15
0.66	5.72
0.75	6.00
0.90	6.40
1.00	6.65
1.50	7.50
2.00	8.10

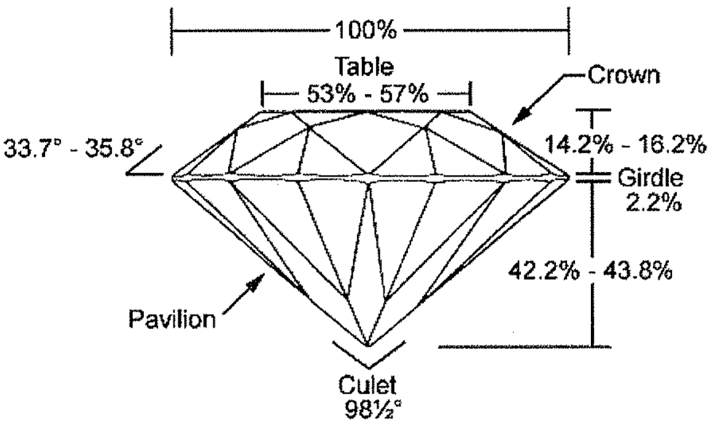
1. <http://www.jewellerycatalogue.co.uk/diamonds/carat_weight.php> accessed on 5th of January 2009

A. 1. Finding the weight of a diamond

1.1 Dimensions of a diamond

The dimensions of a diamond are given in terms of its diameter that we will call D. This diagram shows the 'ideal cut shape' with maximum and minimum percentages of its diameter.

'Ideal cut shape' [2]

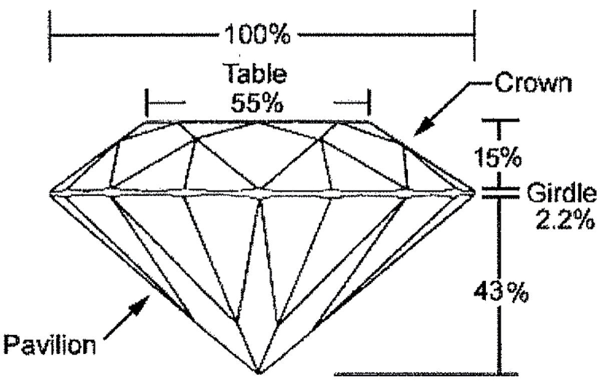


For example:

53%-57% for table mean that the diameter of the table is between 0.53D and 0.57D.

This is the version that I modified as one can notice I took the average of the minimum and maximum percentages.

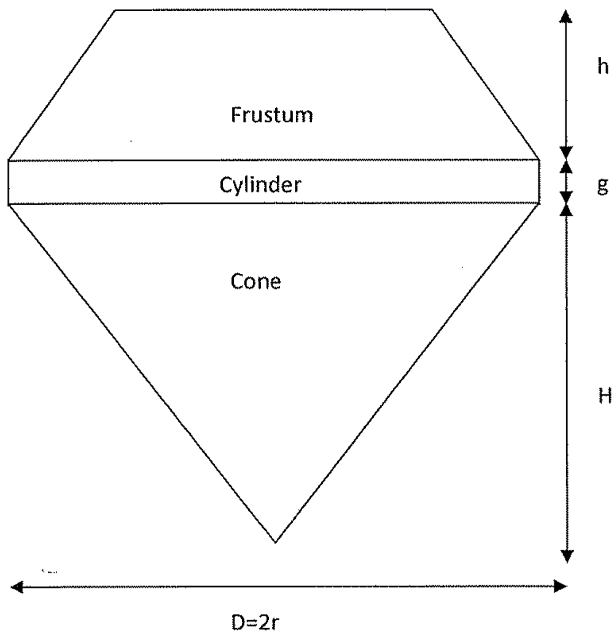
'Ideal cut shape' modified by me



2 <http://www.diamonds11.com/images/ideal_cut_measurements.gif> accessed on the 1st February 2009

1.2 Dividing diamond into simple solids

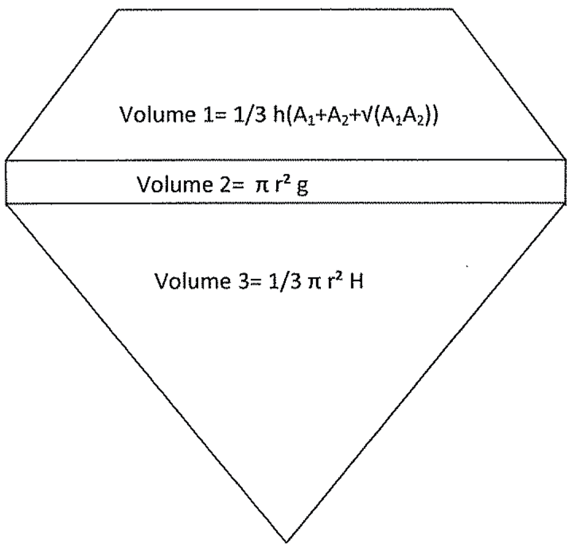
Dividing diamond into solids



In order to find the volume of a diamond, I divided the diamond in three 3dimensional solids and assumed they have shapes of:

1. Frustum
2. Cylinder
3. Cone

Volumes of the solids

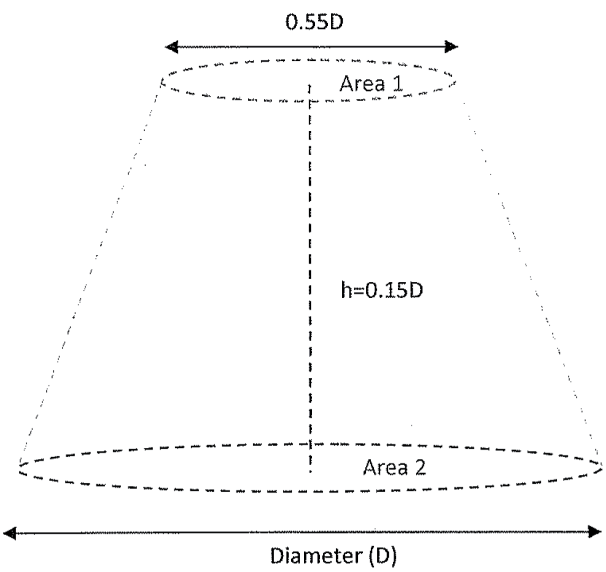


This diagram to the left shows the solid and the formula for each of their volume. The formulas for the volumes of a cone and a cylinder are well known but the formula for the volume of a frustum is not that common. There are many variations of that formula. I choose to use one I have found through research.[1]

Volume 1= $1/3\ (A_1+A_2+\sqrt{(A_1A_2)})$ where A_1 and A_2 are areas of the upper and lower base of the frustum.

3 <http://mathworld.wolfram.com/ConicalFrustum.html> accessed on 27th December 2008

C3



Drawing 1 frustum

1.3 Volume of the crown

This is the formula for the frustum's volume
$V1 = 1/3\ h(A_1+A_2+\sqrt{(A_1A_2)})$

According to the diagram

$A_2 = \pi\ (D/2)^2$

$A_1 = \pi\ (0.55D/2)^2$

$h = 0.15D$

Hence knowing the diameter of a diamond, the volume of the crown can be found.

For example:

If D=3mm=0.3cm then

$A_2 = \pi(0.3/2)^2 = 0.0707$

$A_1 = \pi(0.55(.3)/2)^2 = 0.0214$

$h = 0.15 \bullet 0.3 = 0.165$

$V1 = 1/3 \bullet 0.165 \bullet (0.0214+0.0707+\sqrt{(0.0214+0.0707)})$

$V1 = 0.002cm^3$

Note: The millimetres are converted into cm in order to simplify further calculations

Similar calculations have been done using Excel software for all diameters; these are presented in Table 2 below.

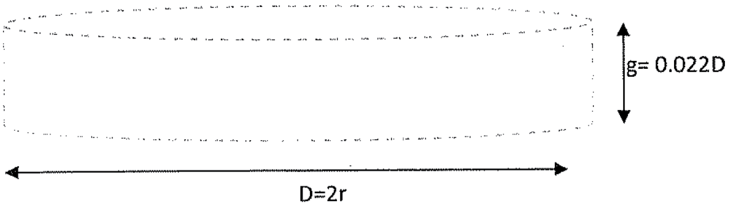
B3

Table 2 Volume of frustum

Diameter					
mm	cm	Area 2 cm²	D• .55cm	Area 1 cm²	V1 cm³
1	0.1	0.0079	0.055	0.0024	0.0001
3	0.3	0.0707	0.165	0.0214	0.002
3.85	0.385	0.1164	0.2118	0.0352	0.0042
4.1	0.41	0.132	0.2255	0.0399	0.0051
4.55	0.455	0.1626	0.2503	0.0492	0.0069
4.8	0.48	0.181	0.264	0.0547	0.0082
5.15	0.515	0.2083	0.2833	0.063	0.0101
5.72	0.572	0.257	0.3146	0.0777	0.0138
6	0.6	0.2827	0.33	0.0855	0.0159
6.4	0.64	0.3217	0.352	0.0973	0.0193
6.65	0.665	0.3473	0.3658	0.1051	0.0217
7.5	0.75	0.4418	0.4125	0.1336	0.0311
8.1	0.81	0.5153	0.4455	0.1559	0.0392

C3

1.4 Volume of a griddle



Drawing 2 Cylinder 3D solid

This is the formula for cylinder's volume

V2= $\pi r^2 g$

According to the diagram

V2= $\pi(D/2)^2 \bullet 0.022D$.

Hence knowing the diameter of a diamond the volume of the griddle can be found.

For example:

If D = 3 mm = 0.3cm then

V2= $\pi \bullet (0.3/2)^2 \bullet (0.022 \bullet 0.3)$

V2= 0.0016 cm^3

Similar calculations to the workout above are shown with different diameters in the table below.

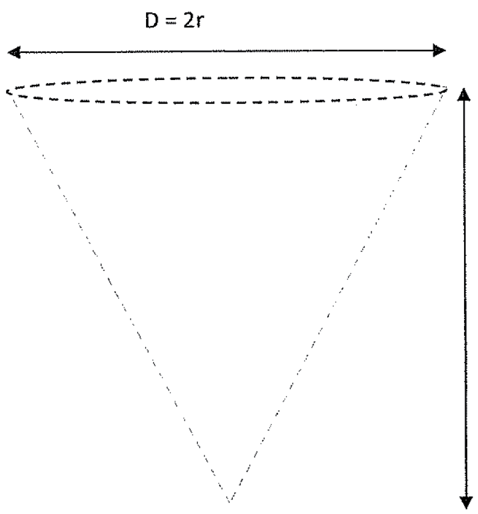
B3

Table 3 Volume of cylinder

Diameter			
mm	cm	Height or griddle	V2 cm^3
1.00	0.100	0.0022	0
3.00	0.300	0.0066	0.0005
3.85	0.385	0.0085	0.001
4.10	0.410	0.009	0.0012
4.55	0.455	0.01	0.0016
4.80	0.480	0.0106	0.0019
5.15	0.515	0.0113	0.0024
5.72	0.572	0.0126	0.0032
6.00	0.600	0.0132	0.0037
6.40	0.640	0.0141	0.0045
6.65	0.665	0.0146	0.0051
7.50	0.750	0.0165	0.0073
8.10	0.810	0.0178	0.0092

Math Project

1.5 The volume of a pavilion



Drawing 3 Cone 3D solid

The formula for the volume of a cone is

V3= 1/3 π r² H

Hence according to the diagram to the left

V3= 1/3• π• (D/2)² • 0.43D

For example:

If D = 3mm = 0.3cm then

V3= 1/3• π • (0.3/2)² • 0.43D

V3=0.003cm³

C3

Similar calculations to the workout above are shown with different diameters in the table below.

Table 4 Volume of cone

Diameter			
mm	cm	Height cm	V3 cm³
1	0.1	0.043	0.0338
3	0.3	0.129	0.003
3.85	0.385	0.1656	0.0064
4.1	0.41	0.1763	0.0078
4.55	0.455	0.1957	0.0106
4.8	0.48	0.2064	0.0124
5.15	0.515	0.2215	0.0154
5.72	0.572	0.246	0.0211
6	0.6	0.258	0.0243
6.4	0.64	0.2752	0.0295
6.65	0.665	0.286	0.0331
7.5	0.75	0.3225	0.0475
8.1	0.81	0.3483	0.0598

B3

1.6 Weight of a diamond

The weight of any object can be found by multiplying its volume by its density. The density of a diamond is known as De= 3.5g/cm³ [5] Volume of a diamond (depending on its diameter) V is sum of V1, V2, and V3 calculated earlier. The weight is obtained by multiplying volume V by density of the diamond.

For example:

If D=3mm = 0.3cm then

$V=V1+V2+V3=0.0066\ cm^3$

$W=V \bullet De$

$W=0.0066cm^3 \bullet 3.5\ g/cm^3$

W=0.023 grams

C3

Similar calculations to the workout above are shown with different diameters in the table below.

Table 5 Workout to find weight of diamond

Diameter cm	Volume 1 cm³	Volume 2 cm³	Volume 3 cm³	V cm³	W g
0.1	0.0001	0	0.0001	0.0002	0.0007
0.3	0.002	0.0016	0.003	0.0066	0.023
0.385	0.0042	0.0026	0.0064	0.0132	0.0462
0.41	0.0051	0.0029	0.0078	0.0157	0.0551
0.455	0.0069	0.0036	0.0106	0.0211	0.0739
0.48	0.0082	0.004	0.0124	0.0246	0.086
0.515	0.0101	0.0046	0.0154	0.03	0.1051
0.572	0.0138	0.0057	0.0211	0.0405	0.1418
0.6	0.0159	0.0062	0.0243	0.0465	0.1626
0.64	0.0193	0.0071	0.0295	0.0559	0.1957
0.665	0.0217	0.0076	0.0331	0.0624	0.2185
0.75	0.0311	0.0097	0.0475	0.0883	0.3091
0.81	0.0392	0.0113	0.0598	0.1103	0.3862

B3

Now I am going to compare my results with those from Table 1. However, the weights in Table 1 are given in carats . So first I have to convert them into grams. Knowing that 1 carat =0.2 grams.[6]

For example:

If weight in carat is 0.05 it is $0.05 \bullet 0.2 = 0.01$grams.

The conversion from carats to grams and comparison with the results obtained by me in Table 5 are presented on page 10 in table 6.

5 <http://encarta.msn.com/encyclopedia_761557986/diamond.html> accessed on 19th of December 2008

6 Gia. "Carat Weight and Size." Diamond Grading Lab manual (2007): 141.

1.7 Comparison between table 1 and result found through mathematical method

To check the accuracy of my results using excel I have graphed the two set of results, the ones I found and the one I got from researching, in the form of a scatter diagram.

Graph 1 Comparison of results

G1

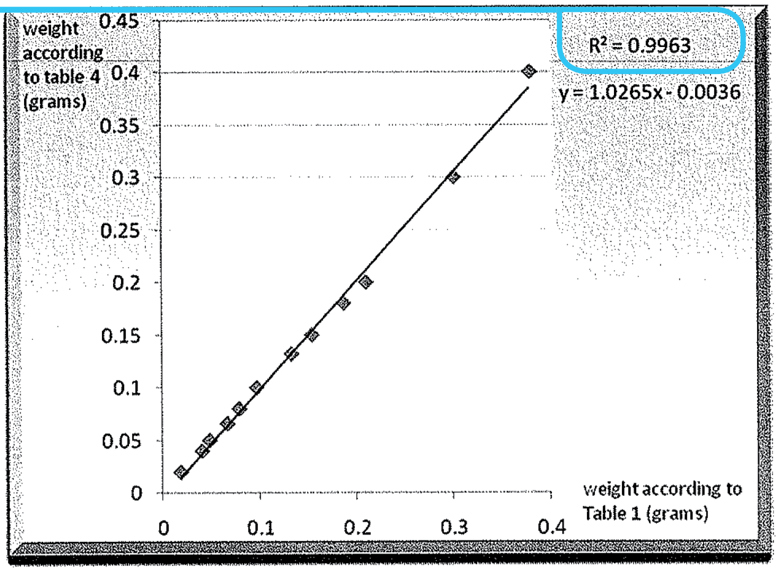


Table 5 Comparison between my results and table

C3

Weights from table 1		Weight from table 5
Carats	Grams	Grams
0.05	0.01	0.0007
0.1	0.02	0.023
0.2	0.04	0.0462
0.25	0.05	0.0551
0.33	0.066	0.0739
0.4	0.08	0.086
0.5	0.1	0.1051
0.66	0.132	0.1418
0.75	0.15	0.1626
0.9	0.18	0.1957
1	0.2	0.2185
1.5	0.3	0.3091
2	0.4	0.3862

To analyse how accurate my results were I used the correlation between the two sets of numbers. The closer the correlation coefficient is to 1 the more accurate my result was. It occurred that the correlation coefficient is .9963 which is surprisingly close to 1.

G1

I also found the line of the best fit. The expected result should be y=x. I have taken 3D solids not the real shape of a diamond the real shape has various facets so it is not straight surface. Nevertheless, my results show that I am very close to the expected result because the equation I got is y=1.0187x-0.0099. This result almost gives y=x, which again proves my results are very close to the numbers given in table 1.

D3

2 Finding a formula for the weight of a polished diamond knowing only the rough weight

2.1 The Cullinan

The other part of the project consisted of looking at a very famous diamond, the Cullinan, which is the world's largest diamond and see how much percent of it was lost while polishing and how much remains. The Cullinan was divided into 9 pieces and their weights(in carats) are given in the table 7. Cullinan 1 is the largest piece polished from the Cullinan they are numbered in order from the largest to the smallest. The third row shows the percentage that the piece is from the initial weight of the rough diamond.

B3

Graph 2 pie chart Cullinan

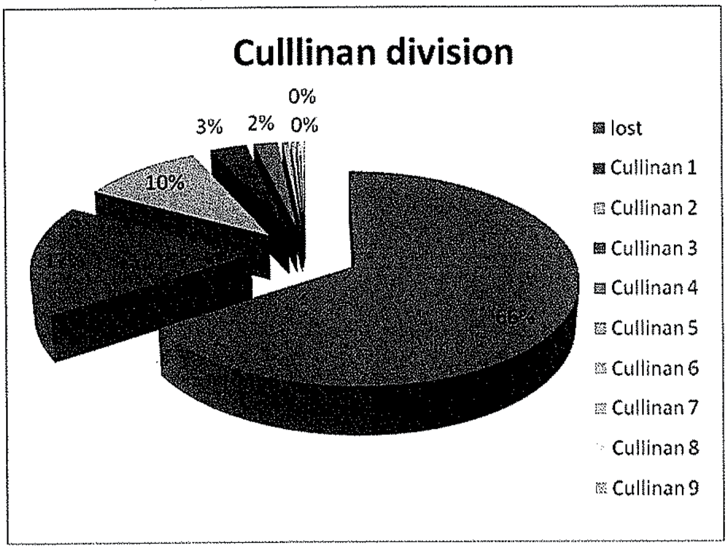


Table 7 Cullinan division

	carats	percentage
Lost	2051	66.0
Cullinan 1	530.2	17.1
Cullinan 2	317.4	10.2
Cullinan 3	94.4	3.0
Cullinan 4	63.6	2.1
Cullinan 5	18.5	0.6
Cullinan 6	11.5	0.4
Cullinan 7	8.8	0.3
Cullinan 8	6.7	0.2
Cullinan 9	4.3	0.1
Total	3106	100%

D3

The data from the table are presented in the form of a pie chart. As the pie chart shows 66% percent of the diamonds initial weight is lost the largest part of the diamond only represents 17% of the diamond's rough weight. While the second largest piece polished only represents 10% of the diamond's weight. I am using this famous example in order to attempt to demonstrate the large mass that is usually lost whilst polishing a rough diamond. All polished pieces give about 34%, so almost 2/3 of the original diamond was lost.

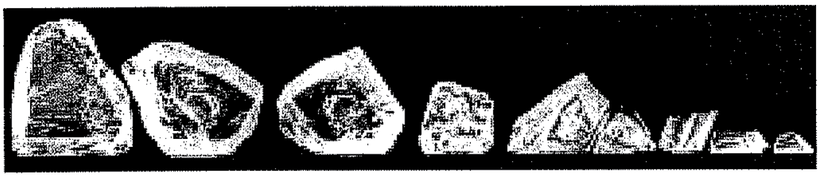


Figure 5 The nine pieces of the Cullinan diamond

2.2 Finding a formula for lost proportion of a diamond

The next step was to look at another set of data that I managed to get from a polishing company 'diamcad'. It shows several rough diamonds that were polished and number of polished diamonds obtained and their weights. The following table represents the diamond initial's weight in the first column. Then, the following columns show the number of stones and their weights. The last two columns show the used and lost percentage of the rough diamond.

Table 8

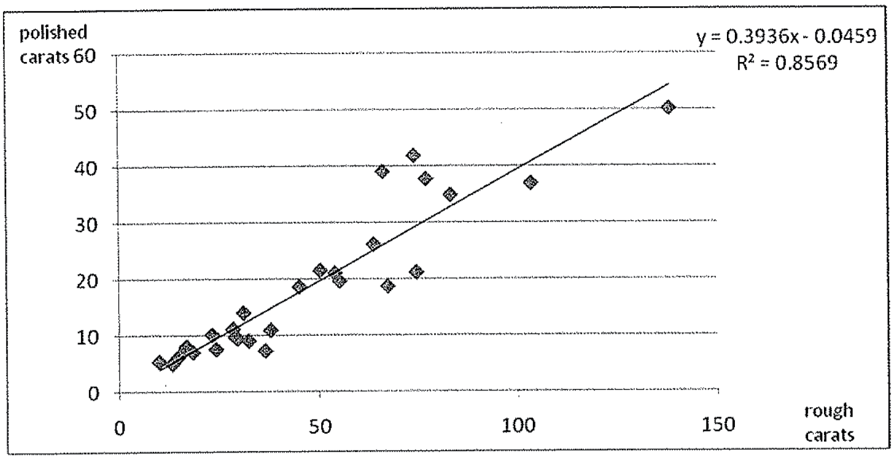
Rough weight (carats)	Weights of polished stones in (carats)									Polished carats	Used %	Lost %
54,07	11,02	2,02	2,01	6,05						21.1	39.02	60.98
31,04	12,01	2,05								14.06	43.3	56.7
29,58	4,35	1,04	1,05	3,02						9.46	31.98	68.02
16,75	6,03	1,02	0,71							7.76	46.33	53.67
67,49	5,08	5,09	4,52	1,01	1,02	1,01	1,02			18.75	27.78	72.22
50,36	11,01	10,55								21.56	42.81	57.19
16,9	4,05	4,02								8.07	47.75	52.25
32,32	7,05	1,02	1,02							9.09	28.13	71.87
18,5	4,05	3,01								7.06	38.16	61.84
10,03	4,01	1,39								5.4	53.84	46.16
23,32	8,24	0,9	1,02							10.16	43.57	56.43
74,23	37,27	1,01	1,09	2,52						41.89	56.43	43.57
37,83	5,06	1,05	0,9	0,93	3,01					10.95	28.95	71.05
66,28	32,24	1,19	5,61							39.04	58.9	41.1
103,87	9,01	10,17	1,05	3,26	10,28	1,01	0,77	1,03	0,38	36.96	35.58	64.42
138,24	26,33	14,02	1,6	2,04	2,01	2,19	2,01			50.2	36.31	63.69
45,09	5,44	2,04	7,05	4,15						18.68	41.43	58.57
63,92	18,6	0,76	4,08	0,74	2,01					26.19	40.97	59.03
55,21	9,05	2,01	2,02	5,56	1,02					19.66	35.61	64.39
14,8	5,11	1,07								6.18	41.76	58.24
36,42	2,03	2,33	1,34	1,59						7.29	20.02	79.98
83,46	20,27	3,22	10,08	1,34						34.91	41.83	58.17
77,23	30,31	5,17	1,38	0,44	0,45					37.75	48.88	51.12
28,4	6,75	2,44	2,01							11.2	39.44	60.56
74,76	10,04	2,01	5,02	3,11	1,04					21.22	28.38	71.62
29,05	4,01	1,58	1,01	3,11						9.71	33.43	66.57
13,34	4,01	1,01								5.02	37.63	62.37
										7.6		

B3

Out of the 29 examples only three of them use more than fifty percent of the initial weight.

The weights of rough and polished stones shown in the table allowed me to draw a scatter diagram.

Graph 3 Comparison of rough and polished weights



Note that the polished is the total mass of all polished diamonds. The equation of the regression line is:

Y=0.4X-0.05

Where Y is the mass of total polished diamonds and X is the mass of the rough diamond. The correlation coefficient is quite high, hence this equation allows me, with good accuracy, predict the mass of polished diamonds when the mass of a rough one is given.

For example the first set :

If X = 54.07

then Y=0.4(54.07)-0.05

Y=21.57 ct

It can be seen in Table 7 that the real results is 21.1 carats. This shows to which extent the formula is accurate.

C3

Page 13

Conclusion

My first goal was to find a mathematical method of finding the weight of a diamond when its diameter is given. I was aware of the fact that the method I took is not perfect and my results are approximations only. It is mostly because of two assumptions I made:

- The dimensions of a diamond are given in an ambiguous way: they have to lie in between certain boundaries. I took for my calculations the average dimensions only.
- The shape of a diamond is more complex than the shape of a frustum, a cylinder and a cone. As a matter of fact no part of a diamond has any of these shapes as the lateral surfaces are not rounded but composed of plane surfaces.

E1

I expected my result to be similar to these found in table 1 but as it occurred, in spite of my assumptions, that they are surprisingly close each other. This was proven by the correlation coefficient. It is worth to mention that all pairs of values in table 5 are close except the first one. It was very confusing and discouraging as this was the first pair of values I compared. Later on, it occurred that the first value in Table 1 was wrong! Hence my calculations allowed me to find a mistake in the table and confirm the remaining values.

D3

One of other advantages of my method over the table is that it allows me to find weight of any diamond including those which are not mentioned in the table.

My second goal was to evaluate the part of a diamond which is wasted after polishing. It occurred that on average it is about 2/3, which is impressively big number.

However, later on, I understood that this powder that originates from the rough diamond being cut does not really vanish as it had its use and was resold. The diamond polishers re-use that powder to polish other diamonds because diamond can only be cut by diamond powder.

I succeeded in finding the formula for the weight of polished diamond (P) if the weight of rough diamond is known (r):

$$P=0.4r-0.05$$

The formula is the equation of the line of the best fit with quite high correlation coefficient, so cannot be used for certain results. Nevertheless it allows to get a good approximation of what polished diamond we can expect. It is worth to add that polishing effects not only a shape but also the effect of polishing diamonds on their colour, the shape that the diamond is polished in affects the colour. For example I have discovered that if one polishes a diamond in a round brilliant shape the diamond enhances one colour grade.

Let me mention that having the data presented in Table 8. I compiled a table of numbers of polished pieces of a diamond weight (see table 9). I believe that further investigation of these data may result with more useful information about polishing diamonds.

Table 9 number of pieces and their weights

Rough weight	0–20	21–40	41–60	61–80	81–100	101–120
n° of stones						
2	5	2	1	0	0	0
3	1	3		1	0	0
4	0	2	2	2	1	0
5	0	1	1	3	0	0
6	0	0	0	0	0	0
7	0	0	0	1	0	1
8	0	0	0	0	0	0
9	0	0	0	0	0	1

It was interesting to see how such areas of mathematics as geometry and statistics allowed me to investigate an area which seems to have more in common with art than science. I was also impressed by finding a mistake in the table. Mathematical accuracy and consistency proved its reliability.

Bibliography

Book
Boonen, Stefan, and Caroline Heens. Diamond a very special story. Amsterdam: Uitgeverij Clavis, 2002.

Encyclopedia
Diamond," Microsoft® Encarta® Online Encyclopedia 2008
http://encarta.msn.com © 1997-2008 Microsoft Corporation. All Rights Reserved.

Journal
Gia. "Carat Weight and Size." Diamond Grading Lab manual (2007): 141.

Websites
<http://www.jewellerycatalogue.co.uk/diamonds/carat_weight.php>.
<http://www.diamonds11.com/images/ideal_cut_measurements.gif>.
<http://www.diamondsourceva.com/Education/FamousDiamonds/famous-diamonds-cullinan.asp>.
<http://mathworld.wolfram.com/ConicalFrustum.html>.