An Investigation to Find the Optimum Dimensions of an Aluminium Drink Can

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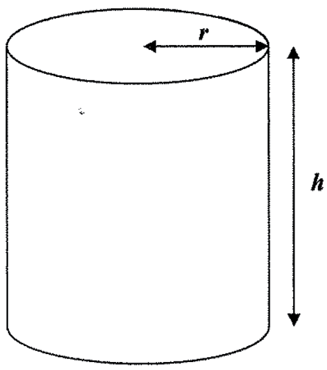

A2

Abstract

In Indonesia, two cola companies, Coca Cola and Pocari Sweat, use different dimensions for their aluminimum cans, with Coca Cola's can having a radius of 5cm and Pocari Sweat's can having a radius of 2cm. Both their cans have the same volume $-330cm^3$. Pocari Sweat's can is narrow than Coca Cola's, so its radius is smaller. The same materials are used to make their cans, with the curved surface of the can being made from cheaper aluminium costing $2Rp/cm^2$, while both the top and bottom is stronger and more expensive at $3Rp/cm^2$.

The aim of my investigation was to discover which company's can is more cost effective, and whether a different design method would be more cost effective than the other. In other words, which is cheaper to produce, while maintaining the same volume? In order to do this, I needed to find the dimensions; radius and height, of an aluminium can which would minimise the cost of producing an aluminium can while keeping a volume of $330cm^3$.

I used three methods to find the dimensions which gave the minimum cost of producing the cans, in order to support my answers and also to see which method is the best to use, in terms of ease and accuracy. The first method I used was a numerical method, in which I first found the areas of the curved surface and top and bottom of the can, allowing me to find the radius which gave the optimum cost of production of the Coca Cola and Pocari Sweat cans. Next, I used an algebraic method to find the radius giving the optimum production cost of the two cans, in which I worked out the formula for the cost of production with the radius as the unknown. Finally, the third and last method I used was a calculus method, where I used the formula for the cost of producing the can that I had found using the algebraic method, and found the gradient function to find its minimum point, which was the minimum cost.

My results found that Coca Cola's 5cm radius can was slightly cheaper than Pocari Sweat's 2cm radius can, but only by marginally, with a can of Pocari Sweat costing 735.4 Rp and a can of Coca Cola costing only 735.24 Rp to produce. There is room for improvement in both cans, however, as having their cans' radiuses at 3.27 cm would allow for the same volume of $330cm^3$ to be stored in them, but at the cheaper production price of 605.27 Rp.

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Part I

I decided my first step would be to find the height of the can in relation to the radius. Full working of the following method in detail is shown on the next page. As a starting point, I found the height of the can when the radius is 5cm. Using the formula $volume = \pi r^2 h$ I could find the height, as the volume and radius were given. I knew the volume of the can was $330cm^3$ and, in this case, the radius is 5cm. By rearranging the formula $volume = \pi r^2 h$ to make height the subject, I found the formula $h = \frac{volume}{\pi r^2}$. From this, I deduced that the height of the can is 4.20cm.

Next, I needed to find the area of the top and bottom of the can would be, given the same dimensions, and what the cost would be to make these parts. Using the formula for area of a circle, $area = \pi r^2$, I found that the area of the top/bottom of the can was $78.5\,cm^2$ separately. This meant that the top of each can would cost 235.5 Rp, as would the bottom.

Then, I had to do the same for the curved part of the can. Using the formula $2\pi r \times h$, I found that the area of the curved surface was $132cm^2$ when the height was 4.2cm and the radius was 5cm. Cheaper aluminium was used for the curved part, costing $2Rp/cm^2$, so this meant that each piece of the curved part cost 264 Rp to produce.

After this, I calculated the total cost of making the can. By adding together the separate costs of producing the top, bottom and curved part of the can, I found that it each can cost 735.2 Rp to produce.

Numerical example:
If the radius of the aluminium can was 5cm…

Area of circle $= \pi r^2$
$= \pi 5^2 = 78.5cm^2$

Volume of can = $\pi 5^2 h$
$= 330 = \pi 5^2 h$
$\frac{330}{\pi 5^2} = h$

Height of can = 4.20cm

Area of top/bottom of can (area of circle) = $78.5cm^2$

Price of top/bottom of can = $78.5cm^2 \times 3Rp$
= 235.6 Rp each

Surface area of curved part = $h \times (\pi d)$
$= 4.2 \times \pi 10$
$= 131.9cm^2$

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Price of curved part of can = $131.9cm^2 \times 2Rp$
$= 263$ Rp

Total cost of making can = price of top + price of bottom + price of curved part
= 235.6 Rp + 235.6 Rp + 263.9 Rp
= 735.1 Rp

Pocari Sweat, uses a can with a smaller radius of 2cm, thus making the can narrower. To see if this design is cheaper to make, I used the same process as above to find the total production cost of one can was used as for calculating Coca Cola can costs.
Once again, the first thing that was required to be found was the height of the can, given that the radius is 2cm. It was found to be 26.3cm.
After this, the area of the top of the can was found to be $12.6cm^2$, so the bottom would be the same.
Next was the calculation of the curved surface. This was found to be $330cm^2$.
Working out the cost of each part was next, with the top and bottom costing 37.8 Rp each and the curved part costing 660 Rp. The total cost of making a Pocari Sweat aluminium can was found to be 735.4 Rp.
This indicated that the Pocari Sweat can was more expensive to produce, but only by a marginal figure, as the Coca Cola can cost 735.2 Rp.

Finally, the minimum cost needed to be found for an aluminium can which has a volume of $330cm^2$, as well as the radius of the can which gives this cost. The values that I had found so far were tabulated, along with the values given by cans with a radius between 0.5cm and 6cm, every 0.5cm.

Formulas used to calculate dimensions of can:
Area of top/bottom of can = πr^2
Height of can = $\dfrac{330}{\pi r^2} = h$
Area of curved surface of can = $h \times (\pi d)$
Total cost of production = 2(area of top + price of bottom) + 3(price of curved part)

Highlighted in red are the dimensions which give the lowest cost of production of the can.

Table 1

Radius	0.5	1	1.5	2	2.5	3	3.5	4	4.5	5	5.5	6
Area of top/bottom	0.7854	3.14159	7.0686	12.5664	19.635	28.274	38.4845	50.265	63.617	78.54	95.033	113.1
Height	420.169	105.042	46.685	26.2606	16.807	11.671	8.57488	6.5651	5.1873	4.2017	3.4725	2.9178
Area of curved surface	1320	660	440	330	264	220	188.571	165	146.67	132	120	110
Total cost of production	2644.71	1338.85	922.41	735.398	645.81	609.65	608.05	631.59	675.04	735.24	810.2	898.58

To demonstrate more clearly the minimum cost as shown by the figures above, I used Microsoft Excel to create a graph, plotting the radius of the can against the total cost of the can.

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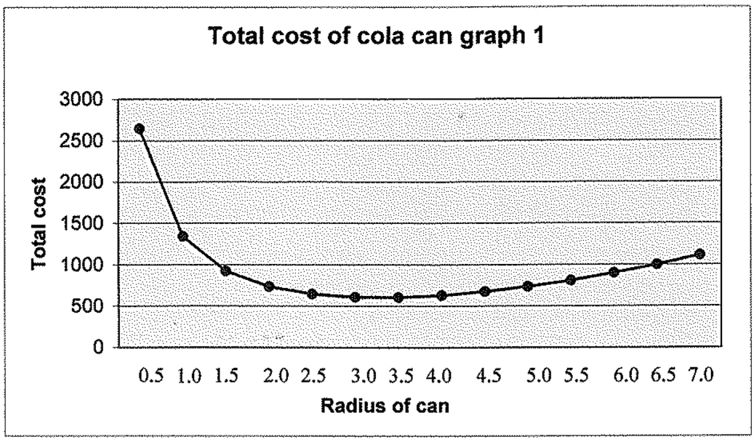

To get a more precise answer, and possibly a lower cost, I calculated the cost of making the can when the radius of it is between 3cm and 3.5cm.

Table 2

Radius	3	3.1	3.2	3.3	3.4	3.5
Area of top/bottom	28.274	30.191	32.17	34.212	36.317	38.4845
Height	11.671	10.931	10.258	9.6458	9.0867	8.57488
Area of curved surface	220	212.9	206.25	200	194.12	188.571
Total cost of production	609.65	606.95	605.52	605.27	606.14	608.05

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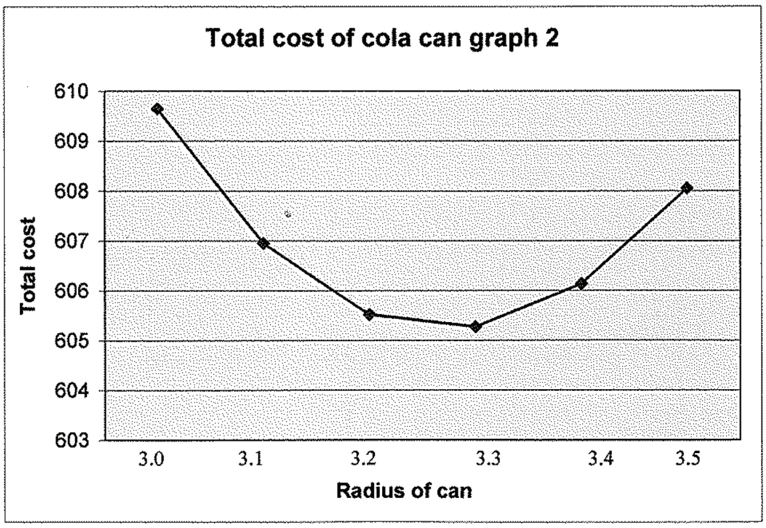
Again, I used a graph to plot the above figures, showing a more precise can radius against the total cost of producing a can with that radius.



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of the can which would allow for the lowest production cost is one with a radius of 3.3cm and a height of 9.6cm, as long as the volume of cola contained in the can is kept at $330cm^3$. With these dimensions, the cost of producing one can would be 605.27 Rp.
The figures in Table 1 show the costs of production of cans with 2cm and 5cm radiuses. In reference to my aim, Coca Cola's can with a 5cm radius is slightly cheaper to produce than Pocari Sweat's 2cm can, though only marginally. A can of Coca Cola costs 735.24 Rp to produce, whereas a can of Pocari Sweat costs 735.4 Rp.

C5

Part II

After completing the numerical method of finding the lowest production cost possible, I wondered if there was another, better way of working it out. I decided my second way of finding the minimum cost of producing the aluminimum can would be algebraically, whereby I had to find a formula which would have the radius as the unknown. This is how I proceeded:

To find the minimum cost of the can, while keeping to the regulation 330 cm^3, I first found the formula for the cost of the both ends of the can put together. The area of a circle = πr^2, and there are two ends, so the area of both is $2 \times \pi r^2$. To get the cost per cm^2, I multiplied the area of the ends by 3, because it costs 3 Rp/cm^2. With the top and bottom of the can each costing 3 Rp/cm^2, I worked out that the cost of both ends together $= 6\pi r^2$.

Next, I found the cost of the curved surface, for which the aluminium costs 2 Rp/cm^2. The area of the curved surface $= 2\pi rh$, and I multiplied it by 2 because the aluminium costs 2 Rp/cm^2. The cost of the curved surface $= 4\pi rh$ if the radius is r and height is h.

After finding these, the formula for the cost of making the whole can is the above mentioned formulas added together so the total cost of making the can $= 6\pi r^2 + 4\pi rh$.

The formula for the minimum cost could only have one variable in it, so I made it the radius. I had to eliminate h and write it in terms of r, in order to have the whole formula in terms of r.

The volume of cola that the can could hold was the only constraint in the investigation, and its formula is:

$v = 330 cm^3 = \pi r^2 h$

I then rearranged the formula to find h:

$$h = \frac{330}{\pi r^2}$$

Because the formula was now in terms of r, I could plug it in to the formula for the total cost of production:

Total cost of producing can $= 6\pi r^2 + \dfrac{4\pi r(330)}{\pi r^2}$

Using this formula, I could substitute in various radius sizes (which would go in the place of 'r'), which would give the result of the lowest production cost for the can.

Another method for finding the lowest production cost of the can while maintaining a volume of 330 cm^3 is by plotting the formula on a graph.

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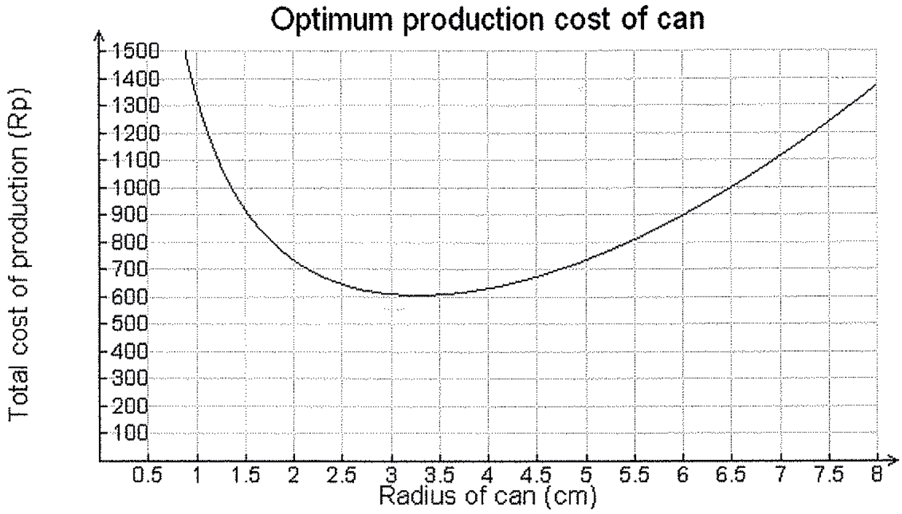

The graph shows that the optimum radius is 3.27cm, as it gives the lowest production cost of 605.27 Rp. This, as well as the costs of the 2cm radius Pocari Sweat can and the 5cm radius Coca Cola can, is in accordance to the answer I achieved from the numerical method in Part I.

D2

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C5

Part III

Although both the previous methods gave the same result, having learnt differential calculus in class, I thought I could validate the methods by using this one to ensure it was not simply due to error or coincidence that they gave the same answer.
Using the formula for the cost of producing the can that I had found using the algebraic method, I could find its gradient function, and from this, find its minimum point, which was the minimum cost and the radius which allowed this cost to be achieved.

Calculus example:
The following is the process I went through to find the radius of the minimum cost:

G1

Total cost of producing can $= 6\pi r^2 + \frac{4\pi r(330)}{\pi r^2}$

I then simplified the formula: $6\pi r^2 + \frac{1320}{r}$

Next I put the simplified formula in index form: $6\pi r^2 + 1320r^{-1}$

I then differentiated the index form of the formula: $\frac{dc}{dr} = 12\pi r - \frac{1320}{r^2}$

I then set the differential equal to zero, which allowed me to solve the formula to find r (the radius). This shows the minimum point, which is the minimum cost, because it shows where the gradient is zero.

$$12\pi r - \frac{1320}{r^2} = 0$$

$$12\pi r = \frac{1320}{r^2}$$

$$\frac{1320}{12\pi} = r^3$$

$$\sqrt[3]{\frac{1320}{12\pi}} = r$$

$$\sqrt[3]{\frac{1320}{12\pi}} = 3.27$$

Using the differential calculus method, the radius giving the optimum production cost is 3.27cm, and by substituting this result into the formula for the total production cost of a can $6\pi r^2 + \frac{4\pi r(330)}{\pi r^2}$ the minimum cost is 605.27 Rp. Both the radius and the minimum production cost are in agreement with the other two methods I used.

E1

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Conclusion and Evaluation

The aim of my investigation was to discover whether Coca Cola's can or Pocari Sweat's can is more cost effective, and whether a different design method would be more cost effective than the other. The three methods I used gave the same answers, so there was no conflict as to the accuracy of them. Coca Cola's 5cm radius can was slightly cheaper than Pocari Sweat's 2cm radius can, but only by a small amount, with a can of Pocari Sweat costing 735.4 Rp and a can of Coca Cola costing only 735.24 Rp to produce. However, both cans can be improved, as having their cans' radiuses at 3.27 cm would allow for the same volume of liquid to be stored in them, but at a cheaper production price.

Each of the three methods was accurate in finding the radius which gave the minimum cost. All three also gave the same answer which strengthens the accuracy and relevancy of the methods as well as the answer itself; a can with a radius of 3.27cm gives the minimum production cost of 605.27 Rp.

It finding the minimum cost using the numerical method was the least confusing, but was more long winded than the algebraic method. The algebraic method was more difficult to work out, mostly due to finding the algebraic formula, but after working out the formula, it was easier to find the minimum cost by creating a graph. The calculus method was fairly simple to solve, and gave the answer in the most straight forward way of the three methods I used, as there was no interpretation of graphs or narrowing down of statistics needed.

This investigation is not totally accurate, as I did not take into account that the aluminium cans are not complete cylinders. Drink cans normally have a dome-like shape in the bottom of the can, so the bottom is not flat, as I have assumed throughout the investigation.

E1

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